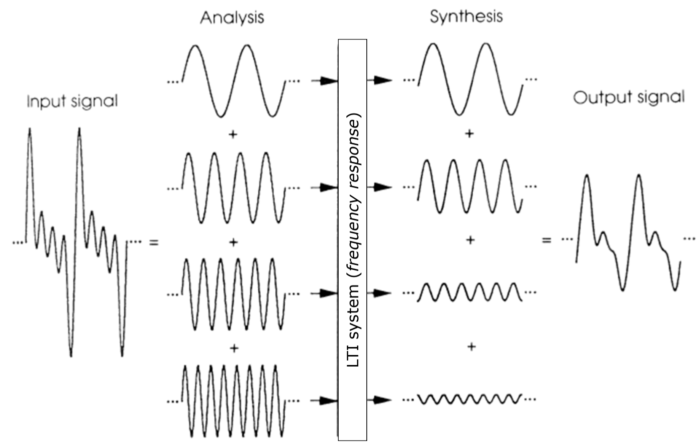
Signals & Systems for Speech & Hearing

Week 8

Impulse responses

The BIG idea



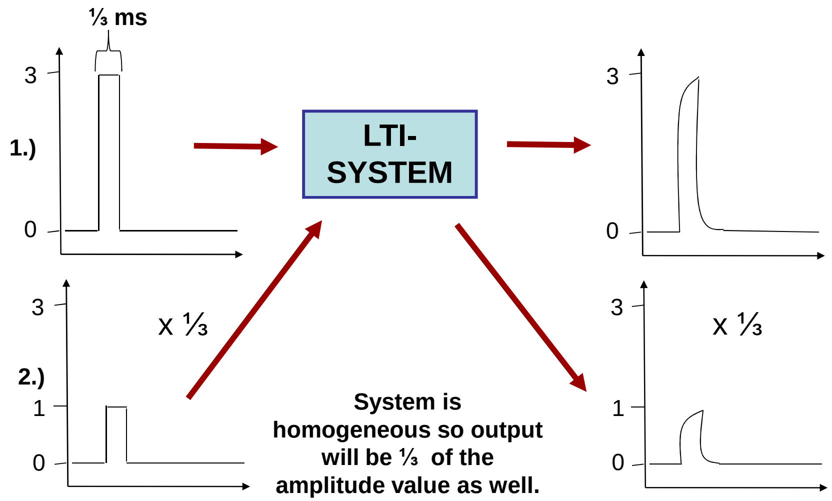
waveform → spectrum — *spectrum → waveform*

Fourier analysis — Fourier synthesis

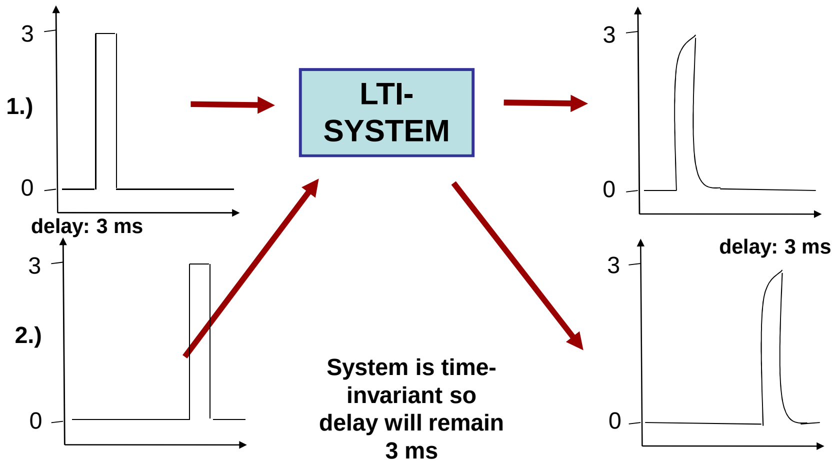
What you know about so far

	time domain (= time on the x axis)	*frequency domain* (= frequency on the x axis)
signals	***waveform***	***spectrum***
systems	***impulse response***	***frequency response***

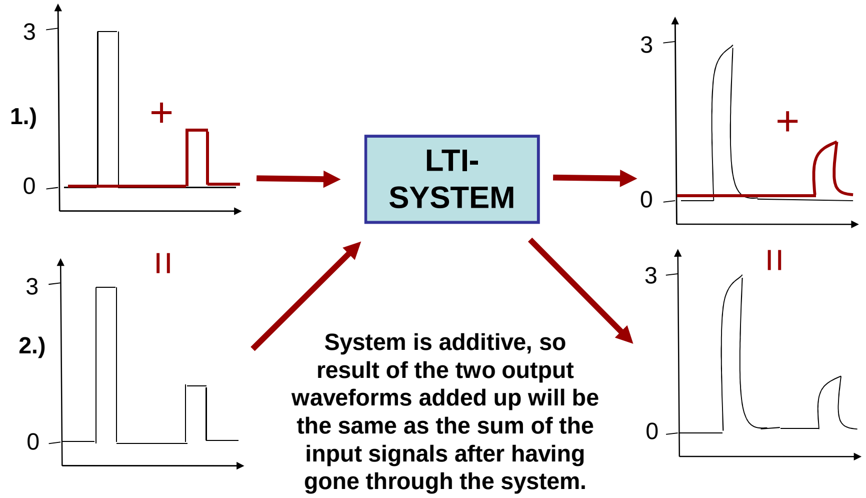
using homogeneity...



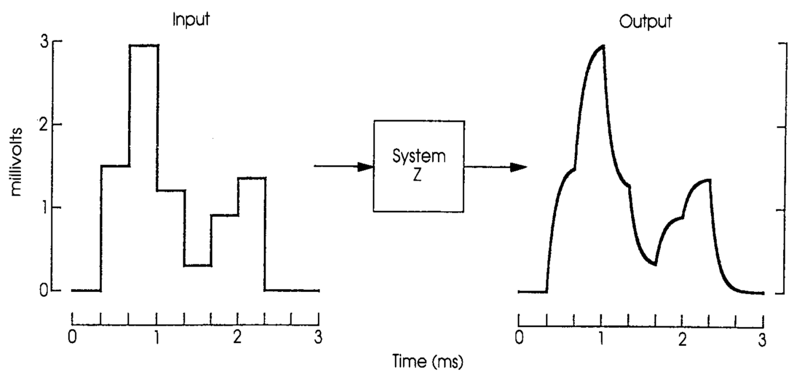
using time-invariance...



using additivity...

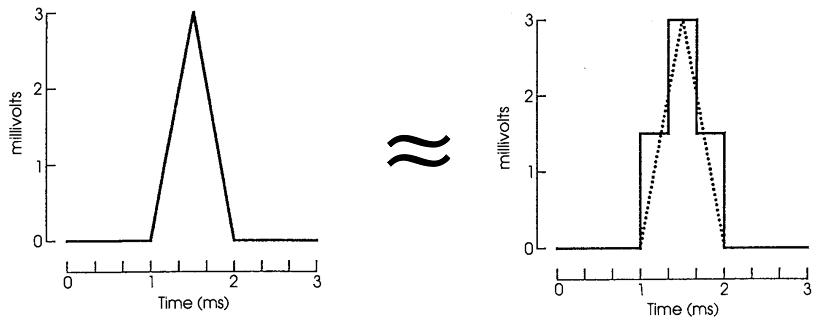


What we can do already

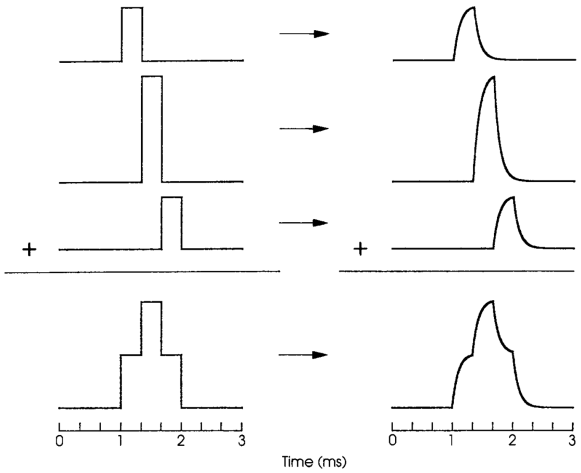


If we know the system response to one single pulse,
we can predict the output for a complicated set of pulses
(constructed by addition of scaled and time-shifted pulses)

Approximation of arbitrary input waves by a combination of rectangular pulses



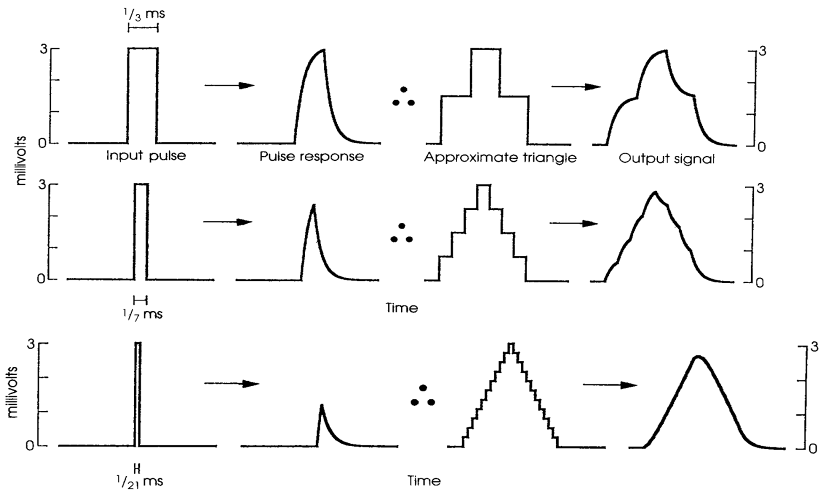
Approximation of arbitrary input waves by a combination of rectangular pulses



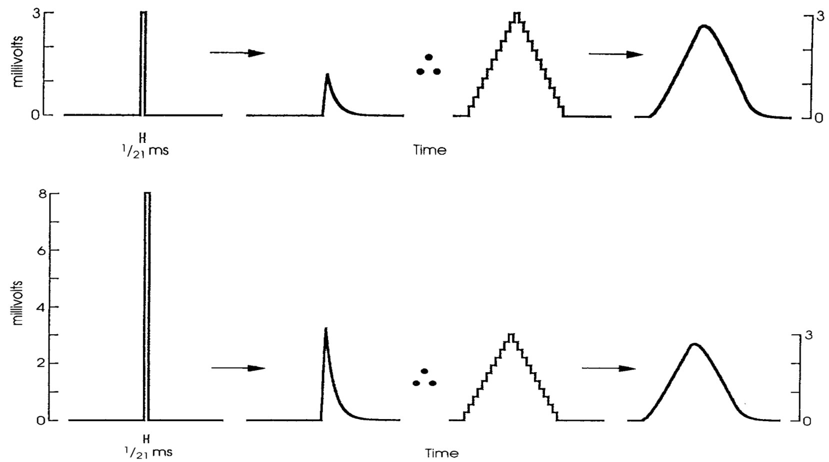
Preliminary summary

- Put a rectangular pulse through the system to see what you get.
- This allows the prediction of system output to:
  - pulses of the same shape but different height
  - pulses of the same shape presented at a different time
  - a sum of an arbitrary number of pulses of the same shape, but any height and at any time.
- If you can adequately approximate a wave by a sum like this, you're done …
  - but this is usually not the case.
- So find a better pulse to approximate waves with.

What would be a better pulse?

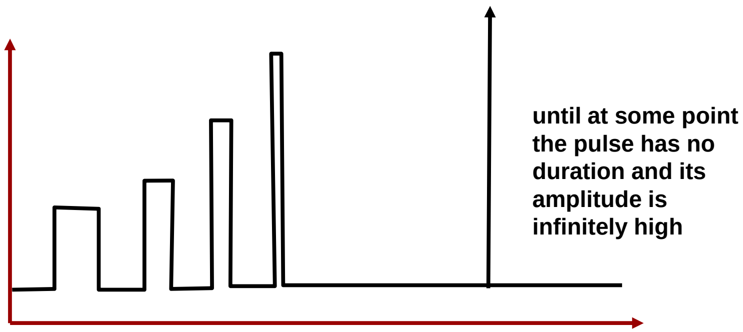


But pulses provide less energy as they become narrower, so output shrinks ...

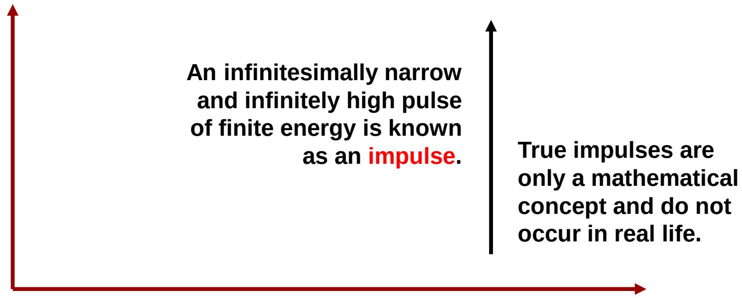


Tending towards an impulse

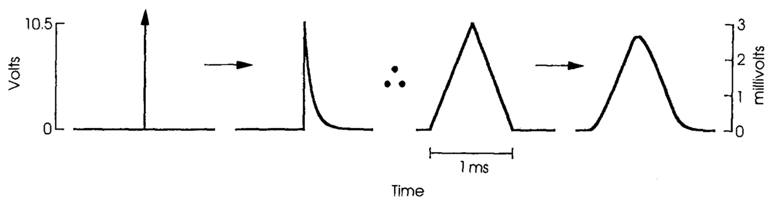
In order keep the output signal of sufficient amplitude, we increase the amplitude of the input pulse as it becomes narrower...



Time-domain characterisation of LTI systems



The best pulse ...



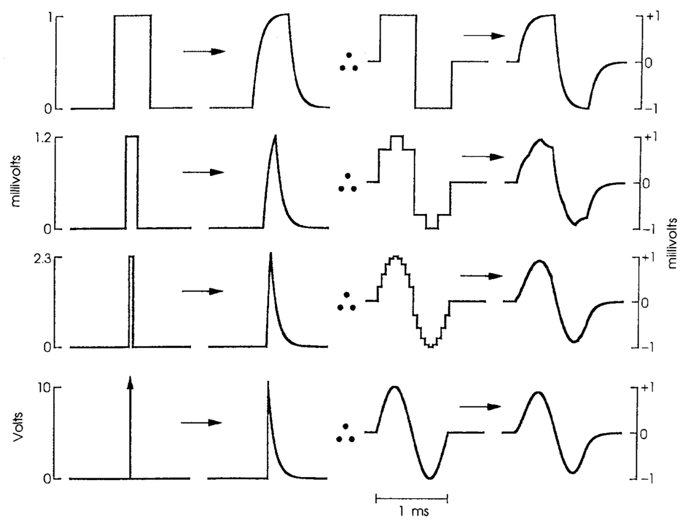
Time-domain characterisation of LTI systems

- Because any wave can be created by adding together an infinite number of impulses appropriately scaled in amplitude and shifted in time ...

- Knowing the impulse response of an LTI system means that the output of the system can be predicted to *any* input.

- Therefore, LTI systems are completely characterised by their impulse response (*time-domain characterisation*).

Another input wave: a single cycle of a sinusoid



What's the relationship between the impulse response and the frequency response?

- Knowing either the impulse response or the frequency response are sufficient to completely characterise a system …
- so they must contain the same information …
- and there might be a simple way to convert between them.

impulse response & frequency response

Suppose we know the frequency response to a system and want to know what its output is to an impulse. Remember how to do that?

Output amplitude (f) = Input amplitude (f) X Amplitude response (f)

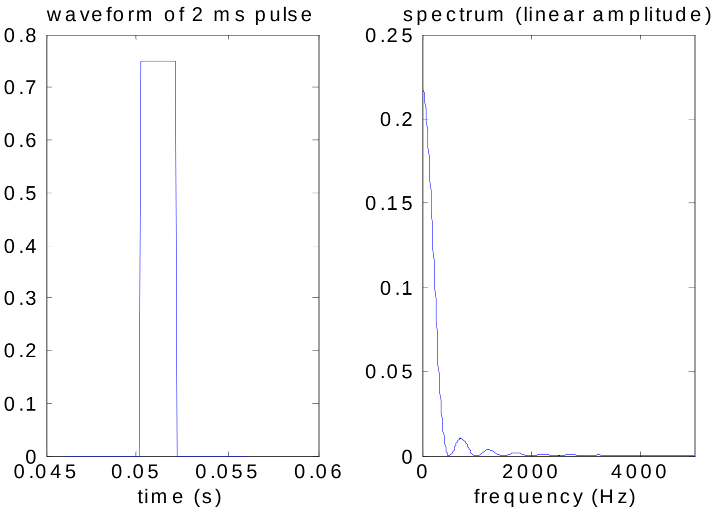
And just for your information:
The same applies for the phase spectrum...

Output phase (f) = Input phase (f) + Phase response (f)

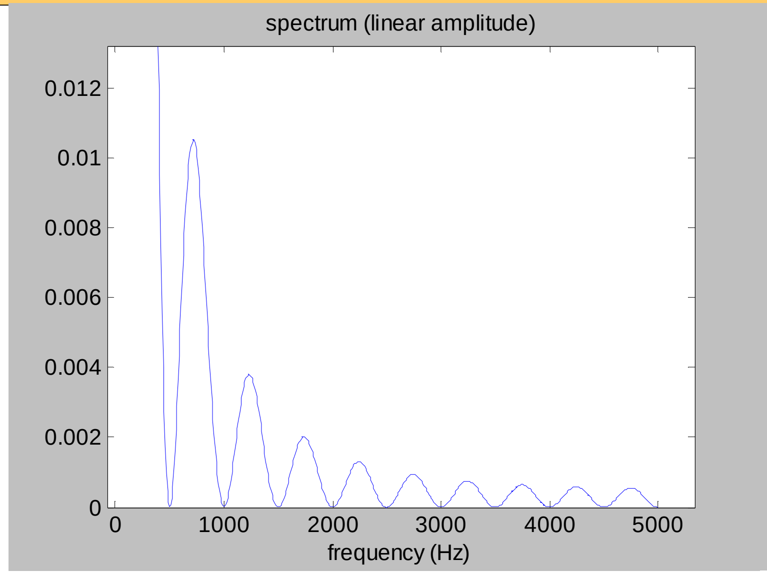
Calculating the spectrum of an impulse directly ...

- is really hard!
- So we'll calculate the spectrum of a rectangular pulse …
- and imagine it getting narrower and narrower.
- Start with a 2 ms pulse

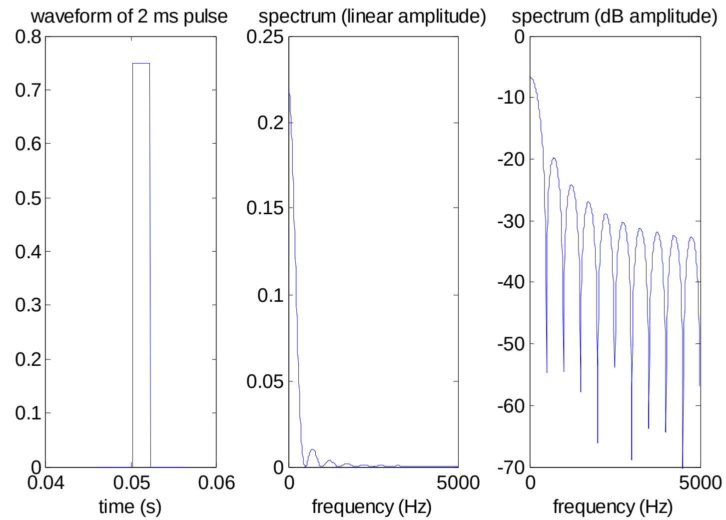
Waveform and spectrum of a 2 ms pulse



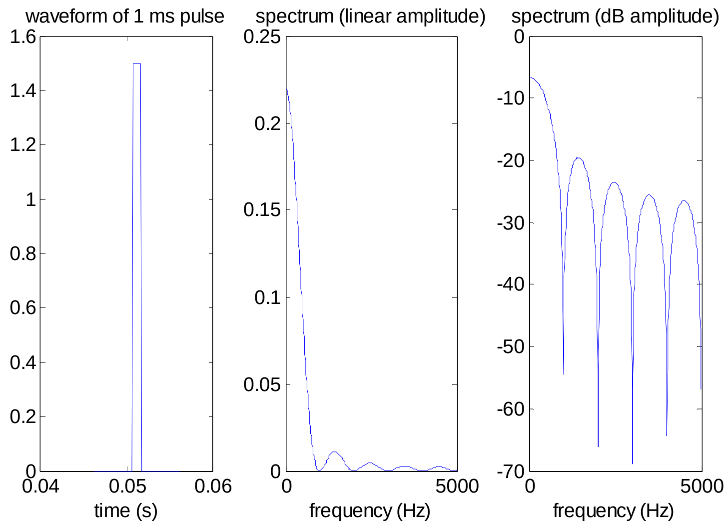
Looking carefully at the spectrum



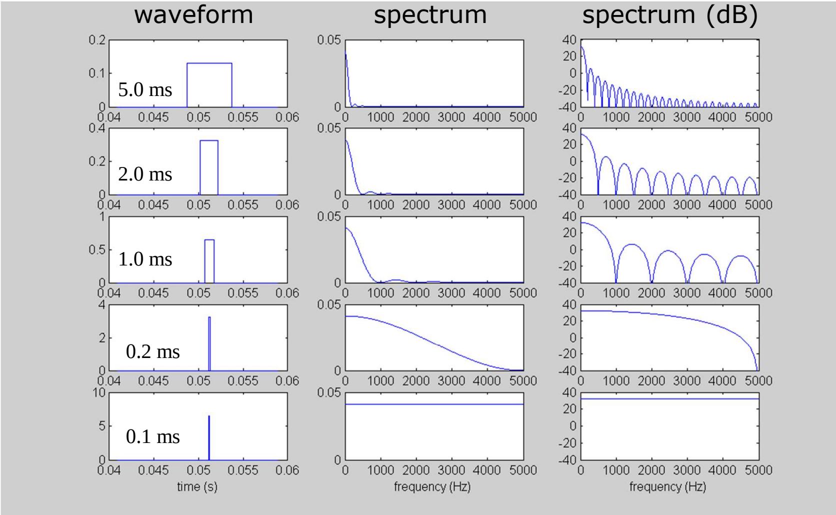
Easier to see spectrum on a dB scale



Amplitude spectrum of a 1 ms pulse



Spectrum of a rectangular pulse (zeros at multiples of 1/pulse duration)



The spectrum of an impulse (infinitely narrow rectangular pulse)

- amplitude spectrum
  - flat (equal amplitude at all frequencies)
  - what other signal has this property?
  - so what else must be different?
- phase spectrum
  - 0 everywhere (when impulse is at time=0)

impulse response & frequency response

Suppose we know the frequency response to a system and want to know what its output is to an impulse. Remember how to do that?

Output amplitude (f) = Input amplitude (f) X Amplitude response (f)

And just for your information:
The same applies for the phase spectrum...

Output phase (f) = Input phase (f) + Phase response (f)

impulse response & frequency response

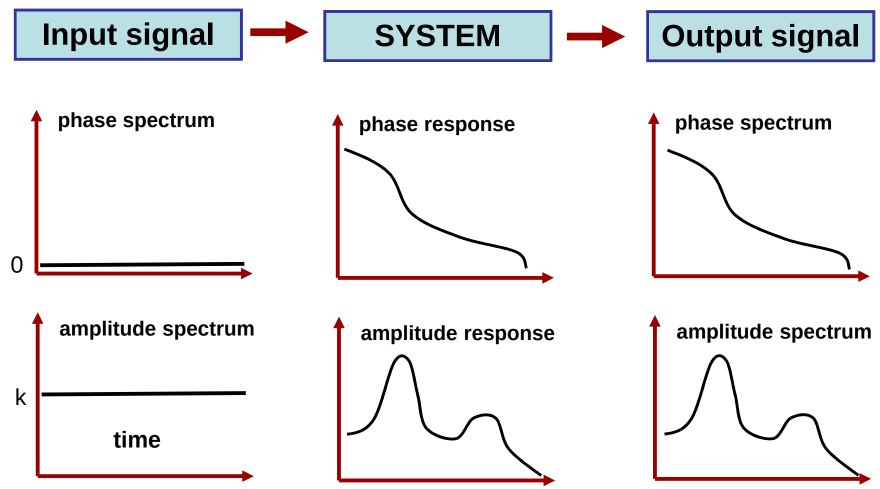
Output amplitude (f) = k x Amplitude response (f)

What is the amplitude spectrum of an impulse?

Answer: constant (k)

→ The amplitude spectrum of a system impulse response is simply the amplitude response of the system.

Response of an LTI system to an impulse



IMPORTANT SUMMARY

The amplitude spectrum of the impulse response is simply the amplitude response of the system.

For your further information:
The phase spectrum of the impulse response is simply the phase response of the system.

Other impulse responses